$$f(x,y) = 2x^3 + 3xy^2$$

$$F_{xy} = \frac{d^2 f}{dy\,dx}$$

$$\frac{df}{dx} = 6x^2 + 3y^2$$

$$\frac{d^2 f}{dx^2} = 12x$$

$$\frac{d^3 f}{dx^3} = 12$$

$$\frac{d^2 f}{d_x d_y} = \frac{d}{dx}\frac{df}{dy}$$

$$\frac{df}{dy} = 6xy$$

$$6xy \frac{d}{dx} = 6y$$

$$\frac{df}{dx} = 6xy$$

$$\frac{d^2 f}{dx^2} = 6x$$

$$\frac{d^3 f}{dy^3} = 0$$

$$\frac{d^2 f}{d_y d_x} = \frac{d}{dy}\frac{df}{dx}$$

$$\frac{df}{dx} = 6x^2 + 3y^2$$

$$6x^2 + 3y^2 \frac{d}{dy} = 6y$$

F_{xy} and F_{yx} don't need to be equal. If there are partial derivatives for every xy then $F_{xy} = F_{yx}$

Laplace

$$u=(x^2+y^2+z^2)^{-\frac{1}{2}} \Rightarrow \frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}=0$$

$$\frac{du}{dx}=2x\cdot\frac{-1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}=-x(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\frac{d^2u}{dx^2}=-(x^2+y^2+z^2)^{-\frac{3}{2}}-x\cdot2x\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}$$

$$\frac{du}{dy}=2y\cdot\frac{-1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}=-y(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\frac{d^2u}{dy^2}=-(x^2+y^2+z^2)^{-\frac{3}{2}}-y\cdot2y\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}$$

$$\frac{du}{dz}=2z\cdot-\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}=-z\cdot(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\frac{d^2u}{dz^2}=-(x^2+y^2+z^2)^{-\frac{3}{2}}-z\cdot2z\cdot\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}$$