Ricatti:

$$R = \frac{dy}{dx} + p(x)y^2 + q(x)y = r(x)$$

if $p(x) = 0$ then R will be Linear

else if $r(x) = 0$ then R will be Bernoulli

Ricatti needs at least one private solution to find homogen solution $y = y_1 + \frac{1}{u}$

$$\frac{du}{dx} - (2y_1 p(x) + q(x))u = p(x) \quad \text{Linear}$$

$$\frac{dy}{dx} + y^2 - (1+2e^x)y = -e^{2x} \qquad y_1 = e^x$$

$p(x) = 1$

$q(x) = -(1+2e^x)$

$r(x) = -e^{2x}$

$$y = e^x + \frac{1}{u}$$

$$\frac{du}{dx} + (-2e^x + 2e^x + 1)u = 1 = \frac{du}{dx} + u = 1 \text{ (Linear)}$$

$$\lambda = e^{\int dx} = e^x$$

$$e^x u = \int e^x dx$$

$$= e^x u = e^x + C$$

$$u = \frac{e^x + C}{e^x}$$

$$y = e^x + \frac{e^x}{e^x + C}$$

$$x^2y' + x^2y^2 + xy = 4 \qquad y_1 = \frac{-2}{x} \qquad y = \frac{-2}{x} + \frac{1}{v}$$

$$y' + y^2 + \frac{1}{x}y = 4$$

$$P(x) = 1$$

$$Q(x) = \frac{1}{x}$$

$$\frac{dv}{dx} - \left(\frac{-4}{x} + \frac{1}{x}\right)v = 1$$

$$r(x) = 4$$

$$= \frac{dv}{dx} + \frac{3}{x}v = 1 \qquad (\text{linear})$$

$$\lambda = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$$

$$x^3 v = \int x^3 \, dx$$

$$x^3 v = \frac{x^4}{4} + C$$

$$v = \frac{x}{4} + C \qquad y = \frac{-2}{x} + \frac{1}{\frac{x}{4} + C} = \frac{-2 + 4C}{x}$$

$x^2y' + x^2y^2 - 2xy = -2$ $\qquad y_1 = \frac{1}{x}$

$y = \frac{1}{x} + \frac{1}{v}$

$y' + y^2 - \frac{2}{x}y = -2$

$p(x) = 1$

$\frac{dv}{dx} - \left(\frac{2}{x} - \frac{2}{x}\right)y = 1$ $\qquad Q(x) = -\frac{2}{x}$

$r(x) = -2$

$\frac{dv}{dx} = 1$

$\frac{dy}{dx} - x^3y^2 + \left(2x^4 - \frac{1}{x}\right)y = x^5$ $\qquad y_1 = x$

$y = x + \frac{1}{v}$

$\frac{dv}{dx} - \left(-2x^4 + 2x^4 - \frac{1}{x}\right)v = -x^3$ $\qquad p(x) = -x^3$

$Q(x) = 2x^4 - \frac{1}{x}$

$\frac{dv}{dx} + \frac{1}{x}v = -x^3$ $\qquad r(x) = x^5$

$\lambda = e^{\int \frac{1}{x}dx} = x$

$xv = \int x \cdot -x^3\, dx = \frac{-x^5}{5} + C$ $\qquad v = \frac{-x^4}{5} + \frac{C}{x}$ $\qquad y = x + \frac{1}{\frac{-x^4}{5} + \frac{C}{x}}$

$$(1-x^3)y' - y^2 + x^2 y = -2x \qquad y_1 = -x^2 \qquad y = -x^2 + \frac{1}{v}$$

$$y' - \frac{1}{(1-x^3)} y^2 + \frac{x^2}{(1-x^3)} y = \frac{-2x}{(1-x^3)}$$

$$P(x) = \frac{-1}{(1-x^3)}$$

$$\frac{dv}{dx} - \left(\frac{2}{1-x^3} x^2 + \frac{x^2}{1-x^3} \right) v = \frac{-1}{(1-x^3)} \qquad Q(x) = \frac{x^2}{(1-x^3)}$$

$$\frac{dv}{dx} - \frac{3x^2}{1-x^3} v = \frac{-1}{(1-x^3)} \qquad \lambda = e^{\int \frac{-3x^2}{1-x^3} dx} \qquad 1-x^3 = u$$

$$-3x^2 dx = du$$

$$\lambda = 1-x^3$$

$$(1-x^3)v = \int (1-x^3) \frac{-1}{(1-x^3)} dx$$

$$= v = \frac{-x+C}{1-x^3} \qquad y = -x^2 + \frac{1-x^3}{-x+C}$$

Orthogonal Trajectories

$x - 4y = c$

$1 - 4\frac{dy}{dx} = 0$

$1 + 4\frac{dx}{dy} = 0$

$\int dy = -4\int dx$

$y = -4x + c$

$y = ce^{-x} \quad c = \frac{y}{e^{-x}}$

$\frac{dy}{dx} = -ce^{-x}$

$\frac{dy}{dx} = \frac{-y}{e^{-x}} \cdot e^{-x}$

$\frac{dx}{dy} = y$

$\int dx = \int y\,dy$

$y = \sqrt{\frac{x + c}{2}}$

$x^2 + y^2 = r^2$

$2x + 2y\frac{dy}{dx} = 0$

$y\frac{-dx}{dy} = -x$

$\int \frac{1}{x}\,dx = \int \frac{1}{y}\,dy$

$\ln x + \ln c = \ln y$

$y = xc$

$y = cx^2 \quad c = \frac{y}{x^2}$

$\frac{dy}{dx} = c2x$

$\frac{dy}{dx} = \frac{2y}{x}$

$\frac{-dx}{dy} = \frac{2y}{x}$

$-\int x\,dx = \int 2y\,dy$

$\frac{-x^2}{2} = y^2 + c^2$

$\frac{-x^2}{2} - c^2 = y^2$

$y^2 = cx^3 \quad c = \frac{y^2}{x^3}$

$2yy' = 3cx^2$

$2yy' = \frac{3y^2}{x}$

$y' = \frac{3y^2}{2x}$

$\frac{-dx}{dy} = \frac{3y}{2x}$

$-\int 2x\,dx = \int 3y\,dy$

$-x^2 = \frac{3y^2}{2} + c$

Clairaut

$p = \frac{dy}{dx}$ $\quad y = xp + f(p)$

$y = xy' + 2y'^2$

$y' = y' + xy'' + 4y'y''$

$p = p + xp' + 4pp'$

$p'(x + 4p) = 0$

$\frac{dy}{dx} = 0 \quad p = c$ general solution

$x + 4p = 0$

$p = \frac{-x}{4}$ write to original

$y = x \cdot \frac{-x}{4} + 2\left(\frac{-x}{4}\right)^2$

$y = \frac{-x^2}{4} + \frac{x^2}{8}$

$y = \frac{-x^2}{8}$

$y = xp + \frac{1}{p}$

$p = p + xp' - \frac{1}{p^2}p'$

$p'\left(x - \frac{1}{p^2}\right) = 0$

$p = c$ general

$x - \frac{1}{p^2} = 0$

$p^2 = \frac{1}{x} \quad p = \frac{1}{\sqrt{x}}$

$y = x \cdot \frac{1}{\sqrt{x}} + \sqrt{x}$

$y = \frac{2x}{\sqrt{x}} \quad y = 2\sqrt{x}$

$y = xp + p - p^2$

$p = p + xp' + p' - 2pp'$

$p'(x - 2p + 1) = 0$

$p = c$

$2p = x + 1$

$p = \frac{x+1}{2}$

$y = \frac{x^2+x}{2} + \frac{x+1}{2} - \left(\frac{x^2}{4} + x + \frac{1}{4}\right)$

$y = xp + p^4$

$p = p + xp' + 4p^3p'$

$p'(x + 4p^3) = 0$

$p = c$

$p^3 = \frac{-x}{4}$

$p = \sqrt[3]{\frac{-x}{4}}$

$y = x \cdot \sqrt[3]{\frac{-x}{4}} + \left(\sqrt[3]{\frac{-x}{4}}\right)^4$

$y = xp + \ln p$

$p = p + xp' + \frac{p'}{p}$

$p'\left(x + \frac{1}{p}\right) = 0$

$x = c$

$x + \frac{1}{p} = 0$

$p = \frac{-1}{x}$

$y = x \cdot \frac{-1}{x} + \ln\left(\frac{-1}{x}\right)$

$y + 1 = \ln\left(\frac{-1}{x}\right)$

$e^{y+1} = \frac{-1}{x}$

$y + 1 = \ln\left(\frac{-1}{x}\right)$

$e^{y+1} = e^{\ln\left(\frac{-1}{x}\right)}$

$y = xp - 2\sqrt{p}$

$p = p + xp' - p^{-\frac{1}{2}} \cdot p'$

$p'\left(x - p^{-\frac{1}{2}}\right) = 0$

$p = c$

$x - p^{-\frac{1}{2}} = 0$

$p = \frac{1}{x^2}$

$y = \frac{1}{x} - \frac{2}{x} = \frac{-1}{x}$

$y = xy' + \sin(y'+1)$

$p = p + xp' + p'\cos(p+1)$

$p'\left(x + \cos(p+1)\right) = 0$

$p = c$

$x + \cos(p+1) = 0$

$\cos(p+1) = -x$

$p+1 = \arccos(-x)$

$p = \arccos(-x) - 1$

$y = (\arccos(-x) - 1) \cdot x + \sin\left(\arccos(-x)\right)$

Method of Undetermined Coefficients

$y'' - 3y' + 2y = e^{5x}$

$m^2 - 3m + 2 = 0$ $\qquad y_h = c_1 e^{2x} + c_2 e^{x}$

$m - 2$
$m - 1$ $\qquad y_p = Ae^{5x}$

$m = 2$ $\qquad y'_p = 5Ae^{5x}$
$m = 1$ $\qquad y''_p = 25Ae^{5x}$

$25Ae^{5x} - 15Ae^{5x} + 2Ae^{5y} = e^{5x}$

$12A = 1$

$A = \frac{1}{12}$ $\qquad y_p = \frac{1}{12}e^{5x}$

$y = y_h + y_p$

$y'' + 3y' + 2y = 4x + 2$

$m^2 + 3m + 2 = 0$
$m = -1$ $\qquad y_h = c_1 e^{-x} + c_2 e^{-2x}$
$m = -2$
$y_p = Ax + B$

$3A + 2Ax + 2B = 4x + 2$ $\qquad y'_p = A$

$A = 2$ $\qquad y''_p = 0$

$B = -2$

$y_p = 2x - 2$ $\qquad y = y_h + y_p$

$y'' - 4y = 10e^{3x}$ $\qquad y_h = c_1 e^{2x} + c_2 e^{-2x}$

$m^2 - 4 = 0$

$m = 2 \quad m = -2$ $\qquad y_p = Ae^{3x}$

$y'_p = 3Ae^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$ $\qquad y''_p = 9Ae^{3x}$

$A = 2$ $\qquad y_p = 2e^{3x}$ $\qquad y = y_h + y_p$

$y'' + 4y = 4x^3$

$y_h = c_1 \cos 2x + c_2 \sin 2x$

$m^2 + 4 = 0$ $\qquad y_p = Ax^3 + Bx^2 + cx + D$

$m = \mp 2i$ $\qquad y'_p = 3Ax^2 + 2Bx + c$

$y''_p = 6Ax + 2B$

$6Ax + 2B + 4Ax^3 + 4Bx^2 + 4cx + D = 4x^3$

$y'' - 4y = 2e^{2x}$ $\qquad y_h = c_1 e^{2x} + c_2 e^{-2x}$

$m^2 - 4 = 0$ $\qquad y_p = Ae^{2x}x$

$m = \mp 2$ $\qquad y'_p = 2Ae^{2x}x + Ae^{2x}$

$y''_p = 4Ae^{2x}x + 2Ae^{2x} + 2Ae^{2x}$

$4Axe^{2x} + 4Ae^{2x} - 4Axe^{2x} = 2e^{2x}$

$A = \frac{1}{2}$ $\qquad y_p = \frac{xe^{2x}}{2}$ $\qquad y = y_h + y_p$

$y''' + 9y' = x\sin x + x^2 e^{2x}$

$m^3 + 9m = 0$ $\qquad y_h = c_1 + c_2 \cos 3x + c_3 \sin 3x$

$m(m^2 + 9) = 0$ $\qquad y_p = Ax\sin x + Bx\cos x + C\sin x + D\cos x$
$+ Ex^2e^{2x} + Fxe^{2x} + ge^{2x}$

$m = 0$
$m = \mp 3i$

$y'' + 4y = \sin x$ $\qquad y_h = c_1 \cos 2x + c_2 \sin 2x$

$m^2 + 4 = 0$ $\qquad y_p = A\sin x + B\cos x$

$m = \mp 2i$

$y'' - y = 2\cos x$ $\qquad y_h = c_1 e^{x} + c_2 e^{-x}$

$m^2 - 1 = 0$ $\qquad y_p = A\cos x + B\sin x$

$m = \mp 1$

$y'' + y = \sin x$ $\qquad y_h = c_1 \cos x + c_2 \sin x$

$m^2 + 1 = 0$ $\qquad y_p = A\sin x + B\cos x \rightarrow Ax\sin x + Bx\cos x$

$m = \mp i$

$y'' + y' = e^{x}\cos x$ $\qquad y_h = c_1 + c_2 e^{-x}$

$m^2 + m = 0$ $\qquad m = 0$ $\qquad y_p = Ae^{x}\cos x + Be^{x}\sin x$
$m(m+1)$ $\qquad m = -1$

Variation of Parameters

$y'' + y = \tan x$ $\quad y_h = c_1\cos x + c_2 \sin x$

$m^2+1=0$ $\quad m = \mp i$ $\quad y_p = c_1 x \cos x + c_2 x \sin x$

$|A| = 1$ $\quad \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \tan x \end{bmatrix}$

$c_1'(x) = \dfrac{\begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix}}{|A|} = \dfrac{-\sin^2 x}{\cos x}$

$c_1(x) = -\int \frac{\sin^2 x}{\cos x}\,dx = -\int \frac{(1-\cos^2 x)}{\cos x}\,dx$

$= -\int \frac{1}{\cos x}dx + \int \cos x\, dx$

$\int -\sec x\, dx \to -\ln|\sec x + \tan x|$ $\quad \sin x$

$c_1(x) = \sin x - \ln|\sec x + \tan x|$

$c_2'(x) = \dfrac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix}}{|A|} = \sin x$ $\quad c_2(x) = \int \sin x\, dx$

$c_2(x) = -\cos x$

$y_p = (\sin x - \ln|\sec x + \tan x|) x \cos x$

$- \cos x \cdot \sin x$

$y = y_h + y_p$

$y'' - 2y' + y = \frac{e^x}{x}$ $\quad y_h = c_1 e^x + c_2 x e^x$

$m^2 - 2m + 1 = 0$ $\quad y_p = c_1(x) e^x + c_2(x) x e^x$

$m=1$

$m=1$

$|A| = (e^x)^2 + x(e^x)^2 - (e^x)^2 \cdot x$ $\quad \begin{bmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{bmatrix}\begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^x}{x} \end{bmatrix}$

$|A| = e^{2x}$

$c_1'(x) = \dfrac{\begin{vmatrix} 0 & xe^x \\ \frac{e^x}{x} & e^x + xe^x \end{vmatrix}}{|A|} = \dfrac{-e^{2x}}{e^{2x}} = -1$ $\quad c_1(x) = -\int 1\, dx$

$c_1(x) = -x$

$c_2'(x) = \dfrac{\begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{x} \end{vmatrix}}{|A|} = \dfrac{1}{x}$ $\quad c_2(x) = \int \frac{1}{x} dx$

$c_2(x) = \ln|x|$

$y_p = -xe^x + \ln|x| x e^x$

$y = y_h + y_p$

$y'' + y = \sec x$

$m^2 + 1 = 0$ $\quad y_h = c_1 \cos x + c_2 \sin x$

$m = \mp i$ $\quad y_p = c_1(x)\cos x + c_2(x) \sin x$

$|A| = 1$ $\quad \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \sec x \end{bmatrix}$

$c_1'(x) = \dfrac{\begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix}}{|A|} = \dfrac{-\sin x}{\cos x}$ $\quad c_1(x) = \int -\frac{\sin x}{\cos x} dx$

$c_1(x) = \ln|\cos x|$

$c_2'(x) = \dfrac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix}}{|A|} = 1$ $\quad c_2(x) = \int 1\, dx$

$c_2(x) = x$

$y'' - 2y' + 2y = e^x \sec x$ $\quad y_h = e^x(c_1 \cos x + c_2 \sin x)$

$m^2 - 2m + 2 = 0$ $\quad y_h = c_1 e^x \cos x + c_2 e^x \sin x$

$m^2 - 2m + 2 - 1 + 1 = 0$ $\quad y_p = c_1(x) e^x \cos x + c_2(x) e^x \sin x$

$(m-1)^2 = -1$ $\quad \begin{bmatrix} e^x \cos x & e^x \sin x \\ e^x\cos x - e^x \sin x & e^x \sin x + e^x \cos x \end{bmatrix}\begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ e^x \sec x \end{bmatrix}$

$m - 1 = \mp i$

$m = 1 \mp i$

$|A| = e^{2x}\cos x \sin x + e^{2x}\cos^2 x - (e^{2x}\sin x \cos x - e^{2x}\sin^2 x)$

$|A| = e^{2x}(\cos^2 x + \sin^2 x) = e^{2x}$

$c_1'(x) = \dfrac{\begin{vmatrix} 0 & e^x \sin x \\ e^x \sec x & \alpha \end{vmatrix}}{e^{2x}} = -\tan x$ $\quad c_1(x) = \int \frac{\sin x}{\cos x} dx$

$c_1 = \ln|\cos x|$

$c_2'(x) = \dfrac{\begin{vmatrix} e^x \cos x & 0 \\ \alpha & e^x \sec x \end{vmatrix}}{e^{2x}} = 1$ $\quad c_2(x) = \int dx$

$c_2(x) = x$

$$y'' + 6y' + 9y = \frac{e^{-3x}}{\sqrt{x^2+1}}$$

$m^2+6m+9=0$ $\quad y_h = c_1 e^{-3x} + c_2 x e^{-3x}$

$m=-3$

$m=-3$ $\quad y_p = c_1(x) e^{-3x} + c_2(x) x e^{-3x}$

$$\begin{bmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & e^{-3x} - 3xe^{3x} \end{bmatrix} \begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^{-3x}}{\sqrt{x^2+1}} \end{bmatrix}$$

$$|A| = e^{-6x}\left(-3x - (-3x)\right) = e^{-6x}$$

$$c_1'(x) = \frac{\begin{vmatrix} 0 & xe^{-3x} \\ \frac{e^{-3x}}{\sqrt{x^2+1}} & \alpha \end{vmatrix}}{|A| = e^{-6x}} = \frac{-x}{\sqrt{x^2+1}}$$

$x^2+1=u$

$2x\,dx = du$

$$c_1(x) = \int \frac{-x}{\sqrt{x^2+1}}\,dx = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\sqrt{x^2+1}$$

$$c_2'(x) = \frac{\begin{vmatrix} e^{-3x} & 0 \\ \alpha & \frac{e^{-3x}}{\sqrt{x^2+1}} \end{vmatrix}}{e^{-6x}} = \frac{1}{\sqrt{x^2+1}}$$

$$c_2(x) = \int \frac{1}{\sqrt{x^2+1}}\,dx = \ln\left|x + \sqrt{x^2+1}\right|$$

$$y''' + y' = \sec x$$

$m^3 + m = 0$ $\quad y_h = c_1 + c_2\cos x + c_3 \sin x$

$m(m^2+1) = 0$ $\quad y_p = c_1(x) + c_2(x)\cos x + c_3(x)\sin x$

$m=0$

$m=\mp i$

$$\begin{bmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 0 & -\cos x & -\sin x \end{bmatrix} \begin{bmatrix} c_1'(x) \\ c_2'(x) \\ c_3'(x) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \sec x \end{bmatrix}$$

$$|A| = \sin^2 x + \cos^2 x = 1$$

$$c_1'(x) = \frac{\begin{vmatrix} 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ \sec x & -\cos x & -\sin x \end{vmatrix}}{1} =$$

$c_1'(x) = \sec x$

$c_1(x) = \int \sec x = \ln|\sec x + \tan x|$

$$c_2'(x) = \frac{\begin{vmatrix} 1 & 0 & \sin x \\ 0 & 0 & \cos x \\ 0 & \sec x & -\sin x \end{vmatrix}}{1} = -1$$

$c_2(x) = -\int 1\,dx$

$c_2(x) = -x$

$$c_3'(x) = \frac{\begin{vmatrix} 1 & \cos x & 0 \\ 0 & -\sin x & 0 \\ 0 & -\cos x & \sec x \end{vmatrix}}{1} = \frac{-\sin x}{\cos x}$$

$$c_3(x) = \int \frac{-\sin x}{\cos x}$$

$c_3(x) = \ln|\cos x|$

Cauchy-Euler

$$ax^2y'' + bxy' + cy = f(x)$$

$x = e^t \qquad t = \ln x \qquad \frac{dt}{dx} = \frac{1}{x}$

$xD = D_1$

$x^2D^2 = D_1(D_1 - 1)$

$x^3D^3 = D_1(D_1-1)(D_1-2)$

$x^2y'' - 3xy' + 3y = 0$

$\perp \qquad x = e^t$

$(x^2D^2 - 3xD + 3)y$

$(D_1(D_1-1) - 3D_1 + 3)y = 0$

$(D_1^2 - D_1 - 3D_1 + 3)y = 0$

$(D_1^2 - 4D + 3)y = 0$

$m^2 - 4m + 3 = 0 \qquad y_h = c_1e^{3x} + c_2e^{x}$

$m-3 \quad m=3$

$m-1 \quad m=1$

$x^2y'' + 3xy' + y = x \qquad x = e^t$

$xD = D_1$

$(D_1(D_1-1) + 3D_1 + 1)y = x \qquad x^2D^2 = D_1(D_1-1)$

$(D_1^2 - D_1 + 3D_1 + 1)y = x$

$(D_1^2 + 2D_1 + 1)y = e^t$

$m^2 - 2m + 1 = 0$

$+1 \qquad y_h = c_1e^{-t} + c_2te^{-t}$

$+1$

$y_p = Ae^t$

$Ae^t + 2Ae^t + Ae^t = e^t \qquad y'_p = Ae^t$

$4A = 1 \qquad y_p = \frac{1}{4}e^t \qquad y''_p = Ae^t$

$A = \frac{1}{4}$

$y_p = \frac{x}{4} \qquad y_h = c_1e^{-\ln x} + c_2te^{-\ln x}$

$y_h = \frac{c_1}{x} + \frac{c_2 \ln x}{x}$

$y = y_h + y_p$

$x^2y'' - 2xy' + 2y = 2\ln x \quad (x>0)$

$x = e^t$

$(x^2D^2 - 2xD + 2)y = 2\ln x \qquad xD = D_1$

$(D_1(D_1-1) - 2D_1 + 2)y = 2\ln e^t \qquad x^2D^2 = D_1(D_1-1)$

$(D^2 - 3D + 2)y = 2t \rightarrow y'' - 3y' + 2y = 2t$

$m^2 - 3m + 2 = 0 \qquad y_h = c_1e^{2t} + c_2e^t$

$m-2 \quad m=2$

$m-1 \quad m=1 \qquad y_p = At + B$

$0 - 3A + 2At + 2B = 2t \qquad y'_p = A$

$2A = 2 \qquad 2B - 3A = 0$

$A = 1 \qquad B = \frac{3}{2} \qquad y''_p = 0$

$y_p = t + \frac{3}{2} \rightarrow \ln x + \frac{3}{2}$

$y_h = c_1x^2 + c_2x$

$y = y_h + y_p$

$x^2y'' - xy' + 4y = \cos(\ln x) \qquad x = e^t$

$(x^2D^2 - xD + 4)y = \cos(\ln x) \qquad xD = D_1$

$x^2D^2 = D_1(D_1-1)$

$(D_1^2 - 2D_1 + 4)y = \cos(t) \rightarrow y'' - 2y' + 4 = \cos(t)$

$m^2 - 2m + 4 = 0 \qquad y_h = e^tc_1\cos\sqrt{3}t + e^tc_2\sin\sqrt{3}t$

$m^2 - 2m + 4 - 3 + 3 = 0 \qquad y_p = A\cos t + B\sin t$

$m^2 - 2m + 1 = -3$

$(m-1)^2 = -3$

$m - 1 = \mp\sqrt{3}i$

$m = 1 \mp \sqrt{3}i$

Reduction of Order

$y = y_1 \cdot v$

$(1-x^2)y'' - 2xy' + 2y = 0 \quad y_1(x) = x$

$y = vx$

$y' = v + xv'$

$y'' = v' + xv''$

$(1-x^2)(v' + xv'') - 2xv - 2x^2v' + 2vx = 0$

$v''(x - x^3) + v'(-2x) + v(2x - 2x) = 0$

$v' = p$

$(x - x^3)\dfrac{dp}{dx} - 2x\,p = 0$