

$$f(x, y) = 2x^3 + 3xy^2$$

$$F_{xy} = \frac{d^2 f}{dy dx}$$

$$\frac{df}{dx} = 6x^2 + 3y^2$$

$$\frac{d^2 f}{dx^2} = 12x$$

$$\frac{d^3 f}{dx^3} = 12$$

$$\frac{d^2 f}{dx dy} = \frac{d}{dx} \frac{df}{dy}$$

$$\frac{df}{dy} = 6xy$$

$$6xy \frac{d}{dx} = 6y$$

$$\frac{df}{dx} = 6xy$$

$$\frac{d^2 f}{dx^2} = 6x$$

$$\frac{d^3 f}{dy^3} = 0$$

$$\frac{d^2 f}{dy dx} = \frac{d}{dy} \frac{df}{dx}$$

$$\frac{df}{dx} = 6x^2 + 3y^2$$

$$(6x^2 + 3y^2) \frac{d}{dy} = 6y$$

F_{xy} and F_{yx} don't need to be equal. If there are partial derivatives for every xy then $F_{xy} = F_{yx}$

Laplace

$$v = (x^2 + y^2 + z^2)^{-\frac{1}{2}} \Rightarrow \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$$

$$\frac{\partial v}{\partial x} = 2x \cdot \frac{-1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -x (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial x^2} = -(x^2 + y^2 + z^2)^{-\frac{3}{2}} - x \cdot 2x \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$\frac{\partial v}{\partial y} = 2y \cdot \frac{-1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -y (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial y^2} = -(x^2 + y^2 + z^2)^{-\frac{3}{2}} - y \cdot 2y \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$\frac{\partial v}{\partial z} = 2z \cdot \frac{-1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} = -z (x^2 + y^2 + z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 v}{\partial z^2} = -(x^2 + y^2 + z^2)^{-\frac{3}{2}} - z \cdot 2z \cdot \left(-\frac{3}{2}\right) (x^2 + y^2 + z^2)^{-\frac{5}{2}}$$

$$z = x^2 \arctan\left(\frac{y}{x}\right) \Rightarrow \frac{\partial^2 z}{\partial x \partial y} \Big|_{(1,1)} = ? \quad \frac{d}{dx} \left(\frac{dz}{dy} \right) = z_{yx}$$

$$\frac{dz}{dy} = x^2 \left(\frac{\frac{1}{x}}{1 + \left(\frac{y}{x}\right)^2} \right) = \frac{x \cdot x^2}{x^2 + y^2} = \frac{x^3}{x^2 + y^2}$$

$$\frac{d}{dx} \left(\frac{x^3}{x^2 + y^2} \right) = \frac{3x^2(x^2 + y^2) - 2x^4}{(x^2 + y^2)^2} = \frac{x^4 + 3x^2 y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} \Big|_{(1,1)} = \frac{1+3}{(1+1)^2} = 1$$

Let's assign Δx as changes for x and Δy for y
 $f(x, y)$

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

$$\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0$$

$$\Delta x \approx dx \quad \Delta y \approx dy$$

$$\Delta z = \frac{df}{dx} \cdot \Delta x + \frac{df}{dy} \cdot \Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

where ε_1 is partial derivative for Δx and ε_2 for Δy

$$\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0$$

$$\Delta z = \frac{df}{dx} dx + \frac{df}{dy} dy + \varepsilon_1 dx + \varepsilon_2 dy$$

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Important part = Differentiation of f

$$z = f(x, y) = x^2 y - 3y \Rightarrow \Delta z = ? \quad dz = ?$$

$$\Delta x = -0.01 \quad \Delta y = 0.02$$

$$\Delta z = [(x + \Delta x)^2 (y + \Delta y) - 3(y + \Delta y)] - (x^2 y - 3y)$$

$$[(x^2 + 2x\Delta x + (\Delta x)^2)(y + \Delta y) - 3(y + \Delta y)] - y(x^2 - 3)$$

$$y + \Delta y (x^2 + 2x\Delta x + (\Delta x)^2 - 3)$$

$$\cancel{yx^2} + \underbrace{y2x\Delta x + y(\Delta x)^2}_{\text{important}} - \cancel{3y} + \underbrace{\Delta y x^2 + \Delta y 2x\Delta x + \Delta y (\Delta x)^2}_{\text{ignored}} - \underbrace{3\Delta y}_{\text{important}} - \cancel{yx^2} + \cancel{3y}$$

$$\underbrace{2xy\Delta x + x^2\Delta y - 3\Delta y}_{\text{important}} + \underbrace{2x\Delta x\Delta y + (\Delta x^2)y + \Delta x^2\Delta y}_{\text{ignored}}$$

$$\Delta z = 2xy\Delta x + (x^2 - 3)\Delta y$$

$$dz = 2xydx + (x^2 - 3)dy \quad \approx$$

As we seen here when Δx and Δy

go to 0 then $dz \approx \Delta z$

$$v = x^2 e^{y/x} \Rightarrow dv =$$

$$dv = \frac{dv}{dx} dx + \frac{dv}{dy} dy \dots$$

$$dx = \left(2x e^{y/x} + \frac{y}{x^2} e^{y/x} x^2 \right) dx$$

$$dy = (x e^{y/x}) dy$$

$$dv = e^{y/x} (2x + y) dx + e^{y/x} dy$$

Chain Rule

$$z = f(x, y) \quad x = g(r, s) \quad y = h(r, s)$$

$$\begin{array}{c} z \\ \swarrow \quad \searrow \\ x \quad y \\ \swarrow \quad \searrow \\ r \quad s \end{array}$$

$$\frac{df}{dr} = \frac{df}{dx} \cdot \frac{dx}{dr} + \frac{df}{dy} \cdot \frac{dy}{dr}$$

$$\frac{df}{ds} = \frac{df}{dx} \cdot \frac{dx}{ds} + \frac{df}{dy} \cdot \frac{dy}{ds}$$

$$z = e^x \sin y, \quad x = \ln t, \quad y = t^2 \quad \frac{dz}{dt} = ?$$

$$\frac{dz}{dx} = e^x \sin y$$

$$\frac{dz}{dy} = e^x \cos y$$

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{1}{t}$$

$$\frac{dy}{dt} = 2t$$

$$e^x \sin y \cdot \frac{1}{t} + e^x \cos y \cdot 2t$$

$$e^x \left(\frac{\sin y}{t} + \cos y \cdot 2t \right)$$

$$e^{\ln t} \left(\frac{\sin(t^2)}{t} + \cos(t^2) \cdot 2t \right)$$

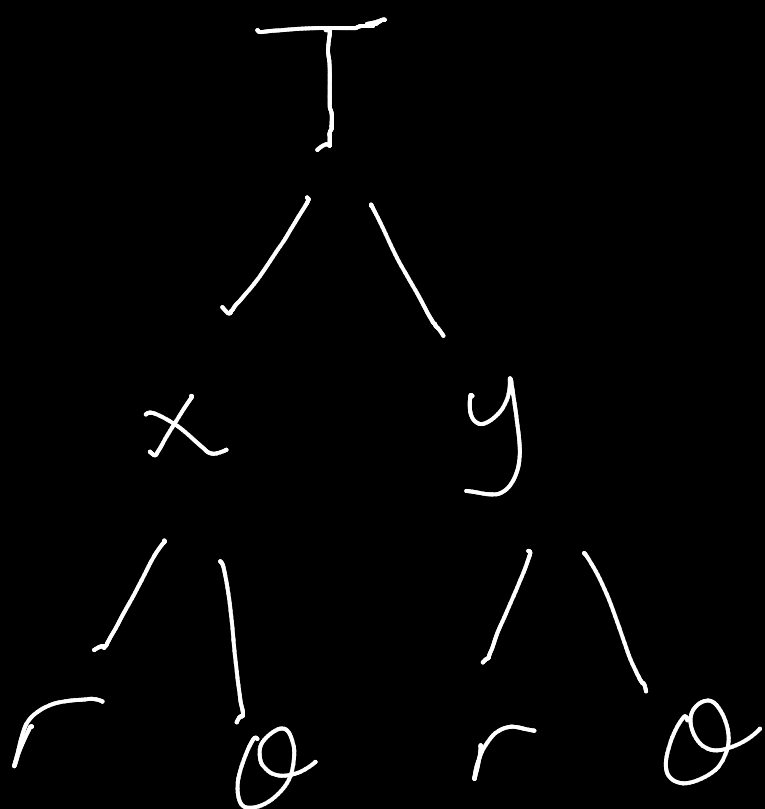
$$\sin(t^2) + 2t^2 \cos(t^2)$$

$$T = x^3 - xy + y^3$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$\frac{dT}{dr} = ?$$

$$\frac{dT}{d\theta} = ?$$



$$\frac{dT}{dr} = \frac{dT}{dx} \cdot \frac{dx}{dr} + \frac{dT}{dy} \cdot \frac{dy}{dr}$$

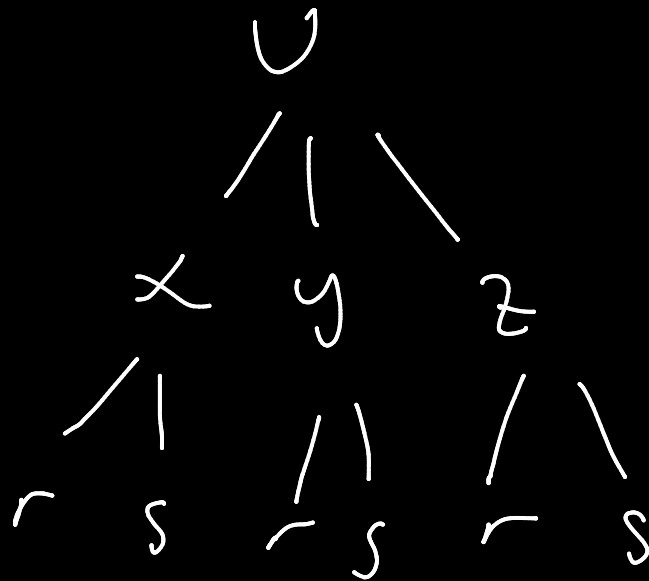
$$\frac{dT}{d\theta} = \frac{dT}{dx} \cdot \frac{dx}{d\theta} + \frac{dT}{dy} \cdot \frac{dy}{d\theta}$$

$$\frac{dT}{dr} = (3x^2 - y) \cos \theta + (3y^2 - x) \sin \theta$$

$$\frac{dT}{d\theta} = (3x^2 - y) \cdot -r \sin \theta + (3y^2 - x) r \cos \theta$$

$$v = z \sin\left(\frac{y}{x}\right), \quad x = 3r^2 + s, \quad y = 4r - 2s^3, \quad z = 2r^2 - 3s^2$$

$$\frac{dv}{ds} = ? \quad \frac{dv}{dr} = ?$$



$$\frac{dv}{ds} = \frac{dv}{dx} \cdot \frac{dx}{ds} + \frac{dv}{dy} \cdot \frac{dy}{ds} + \frac{dv}{dz} \cdot \frac{dz}{ds}$$

$$\frac{dv}{dr} = \frac{dv}{dx} \cdot \frac{dx}{dr} + \frac{dv}{dy} \cdot \frac{dy}{dr} + \frac{dv}{dz} \cdot \frac{dz}{dr}$$